Instructions  Follow instructions carefully, failure to do so may result in points being deducted.

- This homework is one of the ‘honors’ component of this course. It should not take more than two (2) hours to complete. If it does, please let us know.
- The homework must be submitted on paper. Homework neatly formatted in \LaTeX will receive a 10 percent bonus. When formatting in \LaTeX, submit both the .tex and .pdf files via handin, in addition to the hard copy. You will not receive the bonus points if you work with a partner (see below).
- Clearly label each problem and submit answers in order.
- Staple this cover page to the front of your assignment for easier grading.
- Late submissions will not be accepted.
- When you are asked to prove something, you must give a formal, rigorous, and complete a proof as possible. Each step in your proof must contain explanation that would allow us to understand what theorem/logic you have applied to arrive at that step.
- You are to work individually and all work should be your own. Check partner policy below.
- The CSE academic dishonesty policy is in effect (see http://cse.unl.edu/ugrad/resources/academic_integrity.php).

Partner Policy You may work in pairs as long as follow the guidelines below:
1. You must work all problems together. You may not simply partition the work between you.
2. You must use \LaTeX and you may divide the typing duties however you wish.
3. You may not discuss the problems with other groups or individuals.
4. Hand in only one hard copy with both authors’ names.
**Problem A:** Write a CNF formula to model the following scenario:

1. There are four choices of desserts: ice cream, fruit bowl, cake, pie.
2. Exactly one dessert must be selected (i.e., one and only one).

Proceed following the four steps below:

1. First state the propositions and what they represent.
2. State the sentence.
3. Explain the meaning of the clauses.
4. Is the sentence satisfiable? Explain why or why not.

**Problem B:** Write a CNF formula to model the following scenario. (Note that you should only use propositions and no quantifiers):

1. The four states (NE, IA, KS, MO) on the map shown in Figure 1 must be colored using three colors: red, green, and blue.

![Figure 1: Four states (NE, IA, KS, MO)](image)

2. Each state must be colored with exactly one color.
3. Adjacent states (i.e., states sharing a border line) cannot have the same color.

Write a CNF sentence in propositional logic to model this problem.

Hints: You should not use FOL notation or quantified variables, but exclusively propositions. Follow the steps below, which guide you in the process.

1. First state the propositions and what they represent.
   
   (a) For Nebraska, you need to define one atomic proposition for each color. This proposition can be true or false. How many atomic propositions for Nebraska? List all of them.
   
   (b) The problem requires coloring all the states for states. How many atomic propositions you need to define for each state? List all of them.
(c) How many atomic propositions did you need for the problem?

2. State the sentence as a conjunction of disjunctive clauses.

(a) Nebraska must take some color. Write a disjunctive clause stating that fact.

(b) Nebraska cannot take two colors. How many disjunctive clauses do you need to express this fact? Write all these clauses using the propositions you have defined.

(c) Whatever you wrote about Nebraska holds also for each other state. Write all the clauses that state those facts. How many clauses do you need for each state?

(d) How many clauses have you introduced so far?

(e) Consider Nebraska and Iowa. The problem states that they cannot have the same color. Consider the color red. Write a clause that states that Nebraska and Iowa cannot both be red. Repeat the process for each color. How many clauses have you added to this end?

(f) Consider every two neighboring states and repeat the process. How many clauses have you added at this step?

(g) What is the total number of clauses in your sentence modeling this coloring problem?